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Level curvatures, spectral statistics and scaling for interacting particles

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Abstract. The mobility of two interacting particles in a random potential is studied, using the sensitivity of their levels to a change of boundary conditions. The delocalization in Hilbert space induced by the interaction of the two particle Fock states is shown to decrease the mobility in metals and to increase it in insulators. In contrast to the single particle case, the spectral rigidity is not directly related to the level curvature. Therefore, another curvature of topological origin is introduced, which defines the energy scale below which the spectrum has the universal Wigner-Dyson rigidity.

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The interplay between disorder and interactions is a central problem in quantum transport. Recently, it was proposed [1] that interactions could favor delocalization in disordered insulators of large (one particle) localization length L_1. This was first understood for the simple case of two interacting particles (TIP) in a random potential, where a fraction of the TIP-states do have [1] a localization length L_2 larger than L_1. This result is surprising since in disordered metals, repulsive interactions are expected to reduce the conductivity [2] and in disordered insulators, the singularity of the density of states which leads eventually to a gap, also reduces the quantum transport. This delocalization effect can be understood by considering the problem in the Fock space, where the many-body states are delocalized by the interaction. This is a very general idea that we use here in the restricted framework of the two-body problem. By Fock states, we simply mean the 2×2 Slater determinants built out of the one particle states, assuming spinless fermions. These are eigenstates in the absence of interaction, and form a basis of the TIP-Hilbert space. This terminology of Fock states might not be the correct one for this two body problem, but we use it both for brievety and because it will become appropriate for the general N-body problem [3]. We consider bare particles, but the extension to dressed quasi-particles created from the Fermi vacuum is straightforward [4]. In the presence of interactions, characterized by the parameter U, the Fock states are broadened [5,6], and the TIP eigenstates have a finite projection over (typically) $g_2 \equiv \Gamma/\Delta_2^{\mathrm{e}}$ nearby in energy Fock states. Here, Γ is the broadening, characterizing the local density of states in the Fock basis (assuming a Breit Wigner form) and Δ_2^{e} the spacing between the Fock states effectively coupled by U. This delocalization in the Fock basis characterizes both metals and insulators. But the delocalization in the real space occurs only for insulators, since the Fock states are themselves localized in real space.

In order to understand this delocalization, a scaling approach of the TIP-problem was proposed [4] as a generalization of the Thouless block scaling picture. For system sizes $L_1 < L < L_2$, a certain conductance $g_2(L) = \Gamma/\Delta_2^{\mathrm{e}}$ was defined and assumed to obey the usual Ohm's law (*i.e.* the one particle scaling theory of localization) in order to recover the delocalization by interactions. In that work, as in an effective σ model formulation developed [7] later, L_1 was implicitly assumed to be the smallest resolved scale. Therefore, the meaning of g_2 is unclear for $L \ll L_1$ *i.e.* in the metallic regime.

Independently, an analysis [6] of the statistical properties of the TIP-spectrum in the metallic regime has shown that the universal Wigner Dyson rigidity occurs for $g_2 = \Gamma/\Delta_2^{\mathrm{e}}$ consecutive levels, provided $g_2 \geq 1$. In the absence of interaction, the spectrum is essentially of the Poisson type for energies $\Delta_2 \leq E \leq \Delta_1$ where Δ_1 is the one particle mean level spacing. For the one particle spectrum, one knows that Wigner-Dyson rigidity occurs for g_1 consecutive levels, where g_1 is the dimensionless conductance of the non interacting electron gas. For the TIP-case, g_2 plays the role of g_1, as far as the spectral fluctuations are concerned. We will show in this letter that, though the TIP-spectrum becomes more rigid for stronger interactions, the transport in real space is reduced when $L < L_1$.

To that purpose, we describe the transport in terms of the sensitivity of the levels to a change of boundary

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conditions [8]. This is equivalent to closing the system as a ring of length L pierced by a dimensionless Aharonovh-Bohm magnetic flux $\phi = \Phi/\Phi_0$ where $\Phi_0 = hc/e$. We define the two-particle level curvature $C_2(E)$ at energy E as the zero flux curvature of the integrated two-particle density of states $N(E,\phi)$:

$$C_2(E) = \frac{\partial^2}{\partial\phi^2} N(E,\phi)\big|_{\phi=0} \cdot \tag{1}$$

An equivalent expression was shown [9] to describe the d.c residual dimensionless conductance g_1 of the non interacting electron gas under very general conditions set by the random matrix theory. It is therefore natural to try to extend this description to the case of interacting particles.

The main results of this letter are as follows. Assuming a Breit-Wigner form for the local density of states [5,6], we obtain for the two particle curvature C_2 the expression

$$C_2(E) = {C_2}^{(0)}(E) - \frac{g_2}{\Delta_1} I\left(\frac{E}{\Gamma}, \frac{B}{\Gamma}\right) \tag{2}$$

where ${C_2}^{(0)}(E)$ is the value of the curvature in the absence of interactions, B the kinetic energy scale (bandwidth) and $I\,(E/\Gamma, B/\Gamma) \approx g_1\Delta_1/B$ for $g_1 \gg 1$ and of order $-\Delta_1$ for $g_1 \ll 1$. Since $C_2(E)$ fluctuates from sample to sample, let us make more precise what is the meaning of equation (2). For the one particle case, the curvature $C_1(E)$ is characterized, in the metallic regime, by a very broad distribution (generalized Lorentzian) [10]. Its mean value over the statistical ensemble is zero at the center of the band ($E = 0$) so that g_1 is typically given by the width of the distribution, defined for instance by the well behaved quantity $\langle|C_1(E)|\rangle$. $C_2(E)$ should be understood in a similar way *i.e.* as a typical value characterizing the width of a distribution, and not as a mean value. Preliminary numerical results [11] display a similar behavior for the TIP spectrum for a large enough U.

In the metallic regime, the typical values of ${C_2}^{(0)}(E)$ and I are given respectively by $g_1\Delta_1/\Delta_2^{\mathrm{e}}$ and $g_1\Delta_1/B$ so that the total curvature is a decreasing function of the interaction through the term $\propto g_2 g_1$ as expected in a good metal. In the insulating regime ($g_1 \ll 1$), ${C_2}^{(0)}(E)$ is zero up to exponentially small terms while the typical value of I is proportional to $-\Delta_1$ so that C_2 increases with g_2 in agreement with the delocalization effect described above [1].

Although the physical meaning of C_2 is clear, this quantity, unlike the case of non interacting electrons in a random potential, can hardly be considered to be relevant for a scaling theory of the interacting system. From the random matrix theory point of view, the spectrum of the interacting system displays a very unusual behaviour. Usually, the curvatures generated by varying some system parameter also set the energy scale below which the Wigner Dyson rigidity of the spectrum is observed. This is a central point in the scaling theory of localization. The fact that g_2 increases with the interaction in the metallic regime discards C_2 as a candidate for scaling. In other words, we are looking for another "curvature" which would be proportional to g_2 (*i.e.* zero for the non interacting system).

Without interaction ($U = 0$), the TIP-Hamiltonian is separable into two identical one particle Hamiltonians, *i.e.* $H = h_1 + h_2$ with $h_i = \frac{p_i^2}{2M} + V(x_i)$ and where $V(x)$ is the random potential. In other words, the two particles ring can be thought of as two independent one particle rings. For this argument to be valid, we assume either discernable particles, or indiscernable particles with additional quantum numbers. This leads us to associate distinct Aharonov-Bohm fluxes ϕ_1 and ϕ_2 to each ring. The separability of the TIP-Hamiltonian is broken by the interaction $U(x_1 - x_2)$. A variation of ϕ_1 will then induce a current in the second ring characterized by a mutual inductance. One can extend this point of view to the case where a current is driven due to the interactions between two systems, each of them characterized by a well defined gauge field (two SNS junctions for instance). Similarly, this two particle problem in 1d can be thought of as a single particle problem in 2d, where by closing the 2d system on itself as a torus, one can again introduce *two* Aharonov-Bohm fluxes ϕ_1 and ϕ_2. Without interaction, the 2d character of this equivalent one particle model is misleading since the Hamiltonian is separable. But with interactions, we have a genuine 2d system and a topology similar to those considered in the description of the integer quantum Hall effect [12]. We thus define the spectral two-form:

$$C = \mathrm{Im} \sum_A \left\langle \frac{\partial A}{\partial\phi_1} \middle| \frac{\partial A}{\partial\phi_2} \right\rangle \tag{3}$$

where $|A\rangle$ are TIP-eigenstates of energies E_A. Using the structure of the TIP-states in the Fock basis, we obtain:

$$C(\phi_1, \phi_2) = g_2 V(\phi_1, \phi_2) \tag{4}$$

where $V(\phi_1, \phi_2)$ is the Berry connexion whose flux through a closed surface in the flux space is a geometric phase [13]. This relation constitutes the second result of this work. The remainder of this letter is devoted to establish and discuss them.

We consider two particles of mass M interacting through a short range attractive or repulsive potential $U(x_1 - x_2)$ and submitted to a random potential $V(x)$. Defining,

$$\mathcal{L}_2(E) = \frac{M}{2} \sum_{AA'} \delta(E - E_A) \frac{\left|\langle A|P|A'\rangle\right|^2}{E_A - E'_A} \tag{5}$$

where $P = p_1 + p_2$ is the total momentum of the two particles, the curvature $C_2(E)$ defined by equation (1) rewrites:

$$C_2(E) = B\big[\sum_A \delta(E - E_A) + \mathcal{L}_2(E)\big]. \tag{6}$$

We define the one particle states by $(|\alpha\rangle, \epsilon_\alpha)$ and the Fock states by $(|\alpha\beta\rangle, \epsilon_{\alpha\beta} \equiv \epsilon_\alpha + \epsilon_\beta)$. To calculate $C_2(E)$ for the interacting and disordered case, we expand the TIP-states $|A\rangle$ on the Fock states $|\alpha\beta\rangle$:

$$|A\rangle = \sum_{\alpha\beta} C^A_{\alpha\beta} |\alpha\beta\rangle \tag{7}$$

so that $\mathcal{L}_2(E)$ contains a product of four complex amplitudes $C^A_{\alpha\beta}$. To proceed further, we assume as is usual for disordered metals, that only those trajectories which do correspond to time reversed amplitudes contribute to the sum in $\mathcal{L}_2(E)$. Therefore, we keep only terms of the form $\left|{C_{\alpha\beta}}^A\right|^2 \left|{C_{\alpha'\beta'}}^{A'}\right|^2$, *i.e.* those obtained for $\alpha = \gamma, \beta = \epsilon$ and $\alpha' = \gamma', \beta' = \epsilon'$. Combinations of the type $\delta_{\beta\beta'}\langle\alpha'|p_1|\alpha\rangle\delta_{\alpha\alpha'}\langle\beta'|p_2|\beta\rangle$ cancel since $\langle\alpha|p_1|\alpha\rangle = 0$ for zero magnetic flux. Then, $\mathcal{L}_2(E)$ rewrites:

$$\mathcal{L}_2(E) = \frac{M}{8} \sum_{AA'} \delta(E - E_A) \frac{1}{E_A - E_{A'}} \times \sum_{\alpha\beta} \sum_{\alpha'\beta'} \left|{C_{\alpha\beta}}^A\right|^2 \left|{C_{\alpha'\beta'}}^{A'}\right|^2 \delta_{\beta\beta'} \left|\langle\alpha'|p_1|\alpha\rangle\right|^2.$$

In the previous expression appears the density of states $\rho_{\alpha\beta}(E) = \sum_A \delta(E - E_A)|C^A_{\alpha\beta}|^2$ For the TIP-problem it was shown [5,6] that it is well described by the Breit-Wigner form:

$$\rho_{\alpha\beta}(E) = \rho_{\mathrm{BW}}(E - \epsilon_{\alpha\beta}) = \frac{1}{2\pi} \frac{\Gamma}{(E - \Gamma_0 - \epsilon_{\alpha\beta})^2 + \Gamma^2/4}. \tag{8}$$

The shift Γ_0 is negligible for weak enough U and the width Γ can be estimated using the Fermi Golden rule: $\Gamma = 2\pi\langle H^2_{\mathrm{od}}\rangle/\Delta^{\mathrm{e}}_2$ where $\langle H^2_{\mathrm{od}}\rangle$ is the variance of the non diagonal matrix elements of H in the Fock basis. $1/\Delta^{\mathrm{e}}_2$ is the effective density of Fock states coupled by the two-body interaction. For a lattice model with N sites it is assumed that H_{od} are independent normal variables characterized by a variance of the order of U^2/N^3. This estimate comes from the assumption that the one particle wavefunctions are ergodic, as implied by the $O(N)$ invariance in random matrix theory. This corresponds to the zero mode contribution of a diffusion process (higher modes have been considered [3]). Within the zero-mode approximation, $\Delta^{\mathrm{e}}_2 = \Delta_2 = \Delta^2_1/B$, for $g_1 \ll 1$. Using these approximations we obtain,

$$\mathcal{L}_2(E) = \frac{\pi M}{4} \sum_{\alpha\beta} \rho_{\mathrm{BW}}(E - \epsilon_{\alpha\beta}) \sum_{\alpha'} \frac{\left|\langle\alpha'|p_1|\alpha\rangle\right|^2}{\epsilon_\alpha - \epsilon_{\alpha'}} - \frac{\pi}{2}\Gamma \sum_{\alpha\beta} \rho_{\mathrm{BW}}(E - \epsilon_{\alpha\beta}) \sum_{\alpha'\beta'} \rho_{\mathrm{BW}}(E - \epsilon_{\alpha'\beta'}) \times \delta_{\beta\beta'} \frac{\left|\langle\alpha'|p_1|\alpha\rangle\right|^2}{E - \epsilon_{\alpha'\beta'}}.$$

The first term in $\mathcal{L}_2(E)$ contributes to ${C_2}^{(0)}(E)$, *i.e.* to the two-particle curvature in the absence of interactions. The second part is proportional to the interactions so that $\mathcal{L}_2(E)$ rewrites:

$$\mathcal{L}_2(E) = {\mathcal{L}_2}^{(0)}(E) - \frac{\pi}{2}\frac{\Gamma}{\Delta^3_1} I \tag{9}$$

which defines both ${\mathcal{L}_2}^{(0)}$ and I. The sign of $\mathcal{L}_2(E)$ may fluctuate from sample to sample but not the relative sign between the two terms of the rhs of equation (10). The total two-particle curvature is:

$$C_2(E) = {C_2}^{(0)}(E) - \frac{\pi}{2}\frac{B\Gamma}{\Delta^3_1} I. \tag{10}$$

Using $\Delta_2 = \Delta^2_1/B$ and $g_2 = \Gamma/\Delta_2$, we obtain the equation (2) for $C_2(E)$. As discussed in the introduction, the mean value of the curvature C_2 is zero. We are then interested in either the typical value (when it exists) or in the width of the distribution. For the non interacting case ($\Gamma = 0$), the two-particle energies are given by $E_A = \epsilon_{\alpha\beta}$ and the typical value of the curvature [9] $\frac{\partial^2 E_A(\phi)}{\partial\phi^2}\big|_{\phi=0} \propto g_1\Delta_1$ so that ${C_2}^{(0)} \propto \frac{g_1\Delta_1}{\Delta_2}$. In the same way, replacing $\sum_\alpha$ by $\Delta_1^{-1}\int_{-B}^{B} \mathrm{d}x$, the typical width of I defined by its absolute value is:

$$I_{\mathrm{typ}} = \frac{g_1\Delta^2_1}{B} I(0, \infty). \tag{11}$$

The constant $I(0, \infty)$ is given by the integral

$$\int_{-\infty}^{\infty}\mathrm{d}x \int_{-\infty}^{\infty}\mathrm{d}y \frac{1}{(x+y)^2+1} \int_{-\infty}^{\infty}\mathrm{d}z \frac{1}{(z+y)^2+1}\frac{1}{|z+y|} \tag{12}$$

where I_{typ} is evaluated at the center of the band $E = 0$ and in the limit $B/\Gamma \to \infty$ where the bandwidth is much larger than Γ. In this metallic limit $g_1 \gg 1$, we obtain for the typical two particle curvature:

$${C_2}^{(\mathrm{typ})} \simeq \frac{g_1\Delta_1}{\Delta_2} - g_2\frac{g_1\Delta_1}{B}\,. \tag{13}$$

Because of interactions, C_2 is reduced by a small correction. This agrees with numerical results [14] obtained for N spinless fermions. Since $C_2(E)$ can be interpreted as a measure of the transport along the system, we recover that repulsive interactions decrease the conductivity as expected when $g_1 \gg 1$. It could have been anticipated from the behavior of the TIP-energy levels $E_A(\phi)$. In the absence of interaction ($U = \Gamma = 0$), the TIP-spectrum for $g_1 \gg 1$ has many level crossings. This results from the superposition of two independent spectra (although each

of them follows the Wigner Dyson statistics). A finite interaction U removes these level crossings and therefore reduces their curvature. In contrast, when $g_1 \ll 1$, level crossings are already suppressed by one particle localization and a finite interaction may enhance the flux dependence of $E_A(\phi)$, as numerically observed [15].

The relation (11) has been obtained by assuming a uniform broadening of all the Fock states. This is no longer correct when $g_1 \leq 1$, for scales $L_1 < L < L_2$. There, among the N Fock states, only a fraction N_U ($\sim LL_1$ in 1d) is broadened by the interaction. The remaining part $N_0 \approx N - N_U$ has a negligible broadening for a short range interaction since they do correspond to one particle states localized far away from each other. Then, in order to calculate $C_2(E)$ for $L_1 < L < L_2$, we split the sum in equation (5) into two sums corresponding respectively to N_0 TIP-states close to a single Fock state and N_U TIP-states broadened over g_2 Fock states. The first sum corresponds to states $|A_0\rangle$ which are localized within $L_1 \ll L$. In the Hilbert space of these states, it is always possible to build a well defined position operator [16] $\hat{X}$ even for a ring geometry. Therefore, for these states the f-sum rule is fulfilled *i.e.* $\frac{\partial^2 E_A(\phi)}{\partial \phi^2}\big|_{\phi=0} = 0$ and therefore $C_2{}^{(0)}(E)$ is zero up to terms proportional to $\exp -(L/L_1)$. Then, in the expression of $C_2(E)$ remains only the second class of states for which the previous argument does not hold since they are coupled by the interaction U and extended over a scale $L_2 > L$. These states will still contribute to $C_2(E)$ through the second term in the rhs of equation (10). Since $g_1 \approx 0$ (f-sum rule), the analog of equation (6) for one particle yields for $M/2\big|\langle\alpha'|p_1|\alpha\rangle\big|^2$ a typical value of order Δ_1. Using this in equation (11) and making the replacements $\Delta_2 \to \Delta_2^{\rm e} \equiv B/N_U$ and $\Gamma \to \Gamma(\Delta_2^{\rm e})$ given by the Golden rule with a density $1/\Delta_2^{\rm e}$, we obtain for the N_U interaction-assisted states a typical curvature $C_2 \simeq \frac{\Gamma(\Delta_2^{\rm e})}{\Delta_2^{\rm e}}$ when $L_1 < L < L_2$, *i.e.* a mobility increased by the interaction. For $L > L_2$ this contribution decays itself as $\exp -(L/L_2)$. This is nothing but the result of a decimation of the spectrum where all the N_0 TIP-states not coupled by the interaction do disappear. In this regime, the typical curvature C_2 coincides with the TIP-conductance $g_2(L)$ defined by Imry.

Since C_2 is proportional to g_2 only in the localized regime *i.e.* when the one particle mobility is suppressed by localization, it cannot be considered as an appropriate scaling parameter for the TIP-localization transition. For $L \ll L_1$ (metallic regime), the number g_2 of consecutive TIP-levels exhibiting the universal Wigner-Dyson rigidity increases with U, but remains much smaller than C_2 which is essentially dominated by the one particle kinetic part $g_1\Delta_1/\Delta_2$. In order to get rid of this one particle contribution, we are led to introduce the response to two independent fluxes ϕ_1 and ϕ_2. The simplest generalization of a curvature which does not contain the one particle kinetic part is the crossed product (two-form) defined by equation (3). For $U = 0$, it is straightforward to show that $C = 0$, since $|A\rangle = |\alpha\beta\rangle$ and $\sum_{\alpha\beta}\langle\frac{\partial\alpha}{\partial\phi_1}|\alpha\rangle\langle\beta|\frac{\partial\beta}{\partial\phi_2}\rangle$ is real. This is not true anymore for the interacting case. There, using equation (7) we have:

$$C = {\rm Im}\sum_A \sum_{\alpha\beta\gamma\epsilon} C^{A*}_{\alpha\beta} C^A_{\gamma\epsilon} \left\langle \frac{\partial\alpha}{\partial\phi_1}\middle|\gamma\right\rangle \left\langle\beta\middle|\frac{\partial\epsilon}{\partial\phi_2}\right\rangle, \qquad (14)$$

where $C^A_{\alpha\beta} \approx -i\Gamma((E_A - \epsilon_{\alpha\beta}) - i\Gamma)^{-1}$ (this results from the Lippman-Schwinger equation). The approximation previously considered for the $C^A_{\alpha\beta}$ *i.e.* keeping only the combinations $\alpha = \gamma$ and $\beta = \epsilon$, for the calculation of C_2, gives a real term which does not contribute. However, by taking $\alpha = \epsilon$ and $\beta = \gamma$, and approximating $\sum_A \Gamma((E_A - \epsilon_{\alpha\beta)})^2 + \Gamma^2)^{-1}$ by Δ_2^{-1}, we obtain:

$$C = \frac{\Gamma}{\Delta_2}{\rm Im}\sum_{\alpha\beta} \frac{\left\langle\alpha\middle|\frac{\partial H}{\partial\phi_1}\middle|\beta\right\rangle \left\langle\beta\middle|\frac{\partial H}{\partial\phi_2}\middle|\alpha\right\rangle}{(\epsilon_\alpha - \epsilon_\beta)^2} \qquad (15)$$

where we recognize the expression of the two-form connexion $V(\phi_1, \phi_2)$ (see Eq. (4)) whose integral over a closed surface in the flux space is a geometric phase which is independent of the interaction.

Similar curvatures of topological origin were considered in various contexts. To describe the Mott transition in the one dimensional repulsive $U \geq 0$ Hubbard model, Shastry and Sutherland [17] introduced two "fluxes" coupled respectively to the charge and spin degrees of freedom, for a finite density of fermions (no disorder). The two flux curvature measures in this case the difference between the charge and spin susceptibilities. In the context of semiclassical physics, Robbins and Berry [18] did consider the semiclassical approximation of a spectral two-form similar to those given by equation (3) for situations where the classical limit corresponds either to integrable or chaotic systems. In the integrable case, the semiclassical behaviour is equivalent to the Hannay two-form [19]. For non integrable systems, physical realizations of such a two-form is a debated question. The curvature C we introduced for the TIP-problem might be such an example.

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